

Department of Mathematics, School of Sciences

Syllabus for Ph.D. Admission Eligibility Test

Paper-I: Research Methodology

Research and Types of research: Meaning of Research- Objectives of Research- Motivation in Research. Research methods vs Methodology. Types of research – Descriptive vs. Analytical, Applied vs. Fundamental, Quantitative vs. Qualitative, Conceptual vs. Empirical. Research Process. Criteria of good Research. Research Formulation – Defining and formulating the research problem - Selecting the problem - Necessity of defining the problem - Importance of literature review in defining a problem – Literature review – Primary and secondary sources – reviews, treatise, monographs-patents – web as a source – searching the web - Critical literature review – Identifying gap areas from literature review - Development of working hypothesis. Data Collection and analysis: Execution of the research - Observation and Collection of data - Methods of data collection – Modeling, Mathematical Models for research, Sampling Methods- Data processing and Analysis strategies. Data Analysis with Statistical Packages – Hypothesis-testing, Generalization-and Interpretation.

Paper-II: Core Subjects

- 1. Analysis:** Elementary set theory, finite, countable and uncountable sets Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence. Bolzano Weierstrass theorem, Heine Borel theorem. Continuity, uniform continuity, differentiability, mean value theorem. Sequences and series of functions, uniform convergence. Riemann sums and Riemann integral, Improper Integrals. Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral. Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation. Metric spaces, compactness, connectedness.
- 2. Linear Algebra:** Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices, linear equations. Eigenvalues and eigenvectors, Cayley-Hamilton theorem. Matrix representation

of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic forms.

- 3. Complex Analysis:** Algebra of complex numbers, the complex plane, polynomials, Power series, transcendental functions such as exponential, trigonometric and hyperbolic functions. Analytic functions, Cauchy-Riemann equations. Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem. Taylor series, Laurent series, calculus of residues. Conformal mappings, Mobius transformations.
- 4. Algebra:** Permutations, combinations, pigeon-hole principle, inclusion-exclusion principle, derangements. Fundamental theorem of arithmetic, divisibility in \mathbb{Z} , congruences, Chinese Remainder Theorem, Euler's ϕ -function, primitive roots. Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems. Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain. Polynomial rings and irreducibility criteria. Fields, finite fields, field extensions.
- 5. Ordinary Differential Equations:** Existence and Uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs. General theory of homogenous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green's function.
- 6. Partial Differential Equations:** Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs. Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.
- 7. Numerical Analysis:** Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence. Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods. Finite differences, Lagrange,

Hermite and spline interpolation, Numerical differentiation and integration. Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge- Kutta methods.

- 8. Statistics:** Probability, conditional probability, independent events, Baye's theorem. Random Variable and Probability distributions, Mean and Variance. Binomial, Poisson, Normal distributions. Random sampling, Confidence intervals, testing of hypotheses, goodness of fit. Regression.